Energy Efficient Buildings
Walls Homework

1) (U, G) Find the total thermal resistance R (hr-ft^2-F/Btu) and conductance U (Btu/hr-ft^2-F), including the thermal resistances caused by convection coefficients on the interior and exterior (winter) surfaces, of the following wall.

a) 5-inch "sand and gravel concrete" wall. What fraction of the total thermal resistance is caused by the convection coefficients?
R_{sand} = 5 x 0.08 = 0.40 (hr-ft^2-F/Btu)
R_{hi} = 0.68 (hr ft^2 °F / Btu)
R_{ho} = 0.17 (hr ft^2 °F / Btu)
R = R_{sand} + R_{hi} + R_{ho} = 1.25 (hr-ft^2-F/Btu)
U = 1 / R = 0.8 (Btu / hr-ft^2-°F)
(R_{hi} + R_{ho}) / R = 68%

2) (U, G) Find the total thermal resistance R (hr-ft^2-F/Btu) including the thermal resistances caused by convection coefficients on the interior and exterior (winter) surfaces, of the following wall.

a) 2-inch common brick, 2-inch air space, 8-inch concrete blocks.
$R_{common\ brick}$ = 2 x 0.2 = 0.4 (hr-ft^2-F/Btu)
$R_{air\ space}$ = 1.01 (hr-ft^2-F/Btu)
$R_{concrete\ blocks}$ = 2.18 (hr-ft^2-F/Btu)
R_a = $R_{common\ brick}$ + $R_{air\ space}$ + $R_{concrete\ blocks}$ = 3.59 (hr-ft^2-F/Btu)

b) 2-inch common brick, 2-inch Styrofoam insulation, 8-inch concrete blocks.
$R_{common\ brick}$ = 2 x 0.2 = 0.4 (hr-ft^2-F/Btu)
$R_{styrofoam}$ = 2 x 4.2 = 8.4 (hr-ft^2-F/Btu)
$R_{concrete\ blocks}$ = 2.18 (hr-ft^2-F/Btu)
R_b = $R_{common\ brick}$ + $R_{styrofoam}$ + $R_{concrete\ blocks}$ = 10.98 (hr-ft^2-F/Btu)

c) The average outdoor air temperature is 30 F and the average indoor air temperature is 70 F for 2,500 hours during winter. The area of the walls is 1,200 ft^2. The efficiency of the furnace is 90% and the cost of fuel is \$10 /mmBtu. Calculate the fuel energy savings from insulating the wall.
dQ_a = A (Ti – To) /R_a = 1,200 ft^2 x (70 F – 30 F) / 3.59 hr-ft^2-F/Btu = 13,370.47 Btu/hr
dQ_b = A (Ti – To) / R_b = 1,200 ft^2 x (70 F – 30 F) / 10.98 hr-ft^2-F/Btu = 4,371.58 Btu/hr
Fuel energy saving = (dQ_a – dQ_b) x Time / efficiency x \$10 / mmBtu = (13,370.47 – 4,371.58) Btu/hr x 2,500 hrs / 90% x 10^{-6} mmBtu / Btu x \$10 / mmBtu = \$249.97

3) (U, G) Find the total thermal resistance R (hr-ft^2-F/Btu) including the thermal resistances caused by convection coefficients on the interior and exterior (winter) surfaces, of the following walls.

a) wood bevel siding, ½-inch plywood sheathing, 2x4 wood stud wall (studs located 16 inches apart with no insulation in stud cavity), ½-inch gypsum (typical of old houses in UD ghetto).

A_{stud} / A_T = 0.102

$A_{air\ spaces}$ / A_T = 1 – 0.102 = 0.898

R_{hi} = 0.68 hr-ft^2-F/Btu

$R_{wood\ bevel\ siding}$ = 0.81 hr-ft^2-F/Btu

$R_{plywood\ sheathing}$ = (1/2-inch / 1/4-inch) x 0.31 = 0.62 hr-ft^2-F/Btu

$R_{wood\ stud}$ = 3.625 x 0.833 = 3.02 hr-ft^2-F/Btu

$R_{air\ spaces}$ = 1.01 hr-ft^2-F/Btu

R_{gypsum} = 0.45 hr-ft^2-F/Btu

R_{ho} = 0.17 hr-ft^2-F/Btu

$R_{stud\ path} = R_{hi} + R_{wood\ bevel\ siding} + R_{plywood\ sheathing} + R_{wood\ stud} + R_{gypsum} + R_{ho}$ = 5.75 hr-ft^2-F/Btu

$R_{air\ path} = R_{hi} + R_{wood\ bevel\ siding} + R_{plywood\ sheathing} + R_{air\ spaces} + R_{gypsum} + R_{ho}$ = 3.74 hr-ft^2-F/Btu

$R_a = 1 / [(A_{air\ spaces}/A_T/R_{air\ path}) + (A_{stud}/A_T/R_{stud\ path})] = 1 / (0.898/3.74 + 0.102/5.75)$ = 3.878 hr-ft^2-F/Btu

3a	
2x4 stud with 16" spacing	
Input	
Rho (hr-ft2-F/Btu)	0.170
Rsid (hr-ft2-F/Btu)	0.810
Plywood sheathing thickness (in)	0.500
Rplywood sheathing/0.25in (hr-ft2-F/Btu)	0.310
Stud thickness (in)	3.625
Rstud/in (hr-ft2-F/Btu)	0.833
Air spaces thickness (in)	3.625
Rair spaces (hr-ft2-F/Btu)	1.010
Rgyp (hr-ft2-F/Btu)	0.450
Rhi (hr-ft2-F/Btu)	0.680
Stud spacing (in)	16.000
Stud widin (in)	1.625
Area (ft2)	1200
Ti (F)	70
To (F)	40
Hour (hr)	2500
Efficiency	90%
Price	$ 10
Calcs	
Rstud (hr-ft2-F/Btu)	3.020
Rair (hr-ft2-F/Btu)	1.010
Rt, studpath (hr-ft2-F/Btu)	5.750
Rt, airpath (hr-ft2-F/Btu)	3.740
frame frac	0.102
air frac	0.898
Rt (hr-ft2-F/Btu)	3.878
Ut (Btu/hr-ft2-F)	0.258
dQ (Btu/hr)	9283.9725

3b	
2x4 stud with 16" spacing	
Input	
Rho (hr-ft2-F/Btu)	0.170
Rsid (hr-ft2-F/Btu)	0.810
Plywood sheathing thickness (in)	0.500
Rplywood sheathing/0.25in (hr-ft2-F/Btu)	0.310
Stud thickness (in)	3.625
Rstud/in (hr-ft2-F/Btu)	0.833
Cellulose insulation thickness (in)	3.625
Rins/in (hr-ft2-F/Btu)	3.300
Rgyp (hr-ft2-F/Btu)	0.450
Rhi (hr-ft2-F/Btu)	0.680
Stud spacing (in)	16
Stud widin (in)	1.625
Area (ft2)	1200
Ti (F)	70
To (F)	40
Hour (hr)	2500
Efficiency	90%
Price	$ 10
Calcs	
Rstud (hr-ft2-F/Btu)	3.020
Rins (hr-ft2-F/Btu)	11.963
Rt, studpath (hr-ft2-F/Btu)	5.750
Rt, inspath (hr-ft2-F/Btu)	14.693
frame frac	0.102
ins frac	0.898
Rt (hr-ft2-F/Btu)	12.688
Ut (Btu/hr-ft2-F)	0.079
dQ (Btu/hr)	2837.289

b) Same wall as in (b) expect with cellulose insulation blown into stud cavity.
$A_{insulation} / A_T = 1 – 0.125 = 0.898$
$R_{insulation} = 3.625 \times 3.3 = 11.963$ hr-ft^2-F/Btu
$R_{ins\ path} = R_{hi} + R_{wood\ bevel\ siding} + R_{plywood\ sheathing} + R_{insulation} + R_{gypsum} + R_{ho} = 14.693$ hr-ft^2-F/Btu

$R_b = 1 / [(A_{insulation}/A_T/R_{insulation\ path}) + (A_{stud}/A_T/R_{stud\ path})] = 1 / (0.898/14.693 + 0.102/5.75) = 12.688$ hr-ft^2-F/Btu

c) The average outdoor air temperature is 30 F and the average indoor air temperature is 70 F for 2,500 hours during winter. The area of the walls is 1,200 ft^2. The efficiency of the furnace is 90% and the cost of fuel is \$10 /mmBtu. Calculate the fuel energy savings from insulating the wall.
$dQ_a = A (Ti – To) /R_a = 1{,}200$ ft^2 x (70 F – 30 F) / 3.878 hr-ft^2-F/Btu = 9,283.973 Btu/hr
$dQ_b = A (Ti – To) / R_b = 1{,}200$ ft^2 x (70 F – 30 F) / 12.688 hr-ft^2-F/Btu = 2,837.289 Btu/hr
Fuel energy saving = ($dQ_a – dQ_b$) x Time / efficiency x \$10 / mmBtu = (9,283.973 – 2,837.289) Btu/hr x 2,500 hrs / 90% x 10^{-6} mmBtu / Btu x \$10 / mmBtu = \$179

3c	
Annual saving	$ 179

4) (U, G) Find the total thermal resistance R (hr-ft^2-F/Btu) including the thermal resistances caused by convection coefficients on the interior and exterior (winter) surfaces, of the following walls.

a) wood bevel siding, ½-inch plywood sheathing, 2x4 wood stud wall (studs located 16 inches apart, 3.5 inches of fiberglass batt insulation in the stud cavity), ½-inch gypsum.
$A_{stud} / A_T = 0.102$
$A_{air\ spaces} / A_T = 1 – 0.102 = 0.898$

$R_{hi} = 0.68$ hr-ft^2-F/Btu
$R_{wood\ bevel\ siding} = 0.81$ hr-ft^2-F/Btu
$R_{plywood\ sheathing} = (1/2\text{-inch} / 1/4\text{-inch}) \times 0.31 = 0.62$ hr-ft^2-F/Btu
$R_{wood\ stud} = 3.625 \times 0.833 = 3.02$ hr-ft^2-F/Btu
$R_{fiberglass\ batt\ insulation} = 3.5 \times 3.2 = 11.2$ hr-ft^2-F/Btu
$R_{gypsum} = 0.45$ hr-ft^2-F/Btu
$R_{ho} = 0.17$ hr-ft^2-F/Btu

$R_{stud\ path} = R_{hi} + R_{wood\ bevel\ siding} + R_{plywood\ sheathing} + R_{wood\ stud} + R_{gypsum} + R_{ho} = 5.75$ hr-ft^2-F/Btu
$R_{ins\ path} = R_{hi} + R_{wood\ bevel\ siding} + R_{plywood\ sheathing} + R_{air\ spaces} + R_{gypsum} + R_{ho} = 13.93$ hr-ft^2-F/Btu

$R_a = 1 / [(A_{ins\ spaces}/A_T/R_{ins\ path}) + (A_{stud}/A_T/R_{stud\ path})] = 1 / (0.898/3.74 + 0.102/5.75) = 12.171$ hr-ft^2-F/Btu

G	H
4a	
2x4 stud with 16" spacing	
Input	
Rho (hr-ft2-F/Btu)	0.170
Rsid (hr-ft2-F/Btu)	0.810
Plywood sheathing thickness (in)	0.500
Rplywood sheathing/0.25in (hr-ft2-F/Btu)	0.310
Stud thickness (in)	3.625
Rstud/in (hr-ft2-F/Btu)	0.833
Fiberglass batt insulation thickness (in)	3.500
Rins/in (hr-ft2-F/Btu)	3.200
Rgyp (hr-ft2-F/Btu)	0.450
Rhi (hr-ft2-F/Btu)	0.680
Stud spacing (in)	16
Stud widin (in)	1.625
Area (ft2)	1200
Ti (F)	70
To (F)	40
Hour (hr)	2500
Efficiency	90%
Price	$ 10
Calcs	
Rstud (hr-ft2-F/Btu)	3.020
Rins (hr-ft2-F/Btu)	11.200
Rt, studpath (hr-ft2-F/Btu)	5.750
Rt, inspath (hr-ft2-F/Btu)	13.930
frame frac	0.102
ins frac	0.898
Rt (hr-ft2-F/Btu)	12.171
Ut (Btu/hr-ft2-F)	0.082
dQ (Btu/hr)	2957.788

J	K
4b	
2x6 stud with 24" spacing	
Input	
Rho (hr-ft2-F/Btu)	0.170
Rsid (hr-ft2-F/Btu)	0.810
Plywood sheathing thickness (in)	0.500
Rplywood sheathing/0.25in (hr-ft2-F/Btu)	0.310
Stud thickness (in)	5.625
Rstud/in (hr-ft2-F/Btu)	0.833
Fiberglass batt insulation thickness (in)	5.500
Rins/in (hr-ft2-F/Btu)	3.200
Rgyp (hr-ft2-F/Btu)	0.450
Rhi (hr-ft2-F/Btu)	0.680
Stud spacing (in)	24
Stud widin (in)	1.625
Area (ft2)	1200
Ti (F)	70
To (F)	40
Hour (hr)	2500
Efficiency	90%
Price	$ 10
Calcs	
Rstud (hr-ft2-F/Btu)	4.686
Rins (hr-ft2-F/Btu)	17.600
Rt, studpath (hr-ft2-F/Btu)	7.416
Rt, inspath (hr-ft2-F/Btu)	20.330
frame frac	0.068
ins frac	0.932
Rt (hr-ft2-F/Btu)	18.186
Ut (Btu/hr-ft2-F)	0.055
dQ (Btu/hr)	1979.583

b) wood bevel siding, ½-inch plywood sheathing, 2x6 wood stud wall (studs located 24 inches apart, 5.5 inches of fiberglass batt insulation in the stud cavity), ½-inch gypsum.

$A_{stud} / A_T = 0.068$

$A_{ins\ spaces} / A_T = 1 – 0.068 = 0.932$

$R_{wood\ stud} = 5.625 \times 0.833 = 4.686$ hr-ft^2-F/Btu

$R_{insulation} = 5.5 \times 3.2 = 17.6$ hr-ft^2-F/Btu

$R_{stud\ path} = R_{hi} + R_{wood\ bevel\ siding} + R_{plywood\ sheathing} + R_{wood\ stud} + R_{gypsum} + R_{ho} = 7.416$ hr-ft^2-F/Btu

$R_{ins\ path} = R_{hi} + R_{wood\ bevel\ siding} + R_{plywood\ sheathing} + R_{air\ spaces} + R_{gypsum} + R_{ho} = 20.33$ hr-ft^2-F/Btu

$R_b = 1 / [(A_{insulation}/A_T/R_{insulation\ path}) + (A_{stud}/A_T/R_{stud\ path})] = 1 / (0.932/20.33 + 0.068/7.416) = 18.186$ hr-ft^2-F/Btu

c) The average outdoor air temperature is 30 F and the average indoor air temperature is 70 F for 2,500 hours during winter. The area of the walls is 1,200

ft^2. The efficiency of the furnace is 90% and the cost of fuel is \$10 /mmBtu. Calculate the fuel energy savings from insulating the wall.
dQ_a = A (Ti – To) /R_a = 1,200 ft^2 x (70 F – 30 F) / 12.171 hr-ft^2-F/Btu = 2,957.788 Btu/hr
dQ_b = A (Ti – To) / R_b = 1,200 ft^2 x (70 F – 30 F) / 18.186 hr-ft^2-F/Btu = 1,979.583 Btu/hr
Fuel energy saving = (dQ_a – dQ_b) x Time / efficiency x \$10 / mmBtu = (2,957.788 – 1,979.583) Btu/hr x 2,500 hrs / 90% x 10^{-6} mmBtu / Btu x \$10 / mmBtu = \$27.17

4c	
Annual saving	\$ 27.17

5) (G) Find the total thermal resistance R (hr-ft^2-F/Btu) including the thermal resistances caused by convection coefficients on the interior and exterior (winter) surfaces, of the following walls.

a) XPS-insulated, 6.5-inch thick Structural Insulated Panel with wood bevel siding on the exterior and ½-inch gypsum on the interior.
$R_{xps,6.5\text{-}inch}$ = 29.5 hr-ft^2-F/Btu
$R_{wood\ bevel\ siding}$ = 0.81 hr-ft^2-F/Btu
R_{gypsum} = 0.45 hr-ft^2-F/Btu
R_T = 29.5 + 0.81 + 0.45 = 30.76 hr-ft^2-F/Btu

6) (U, G) Consider a house in Dayton, Ohio with four, 40-ft x 8-ft walls with an overall thermal resistance of 15 hr-ft^2-F/Btu including convection coefficients.

a) Calculate the average winter outdoor air temperature as the average of the average monthly temperatures as determined from TMY3 data by WeaTran for November through March.

Mo	Yr	Ta(F)	Sol-H(Btu/ft2dy)	Sol-E(Btu/ft2dy)	Sol-S(Btu/ft2dy)	Sol-W(Btu/ft2dy)	Sol-N(Btu/ft2dy)	w(lbw/lba)	Tg(F)
01	1995	24.60	571	288	707	321	173	0.0023	45.68
02	1995	26.92	824	427	851	410	214	0.0025	43.10
03	1995	42.48	1164	579	860	548	277	0.0040	42.72
04	1995	51.72	1516	730	809	634	313	0.0052	45.73
05	1995	63.36	1773	808	694	771	395	0.0072	49.54
06	1995	70.31	1947	859	643	786	410	0.0110	53.69
07	1995	75.04	1893	844	680	830	411	0.0139	56.54
08	1995	72.77	1644	773	776	740	365	0.0129	58.79
09	1995	63.64	1381	630	980	638	285	0.0105	59.12
10	1995	51.20	1001	495	1016	492	216	0.0056	56.96
11	1995	40.94	614	334	703	317	191	0.0042	53.58
12	1995	30.85	485	234	666	259	138	0.0032	49.48

Average winter outdoor air temperature:
(40.94+30.85+24.60+26.92+42.48)/5 = 33.158 F

b) Calculate the total heat loss (mmBtu = million Btu) through the walls from November through March assuming the inside air temperature is 72 F.
dQ = A(Ti-To)/R = 4*40*8*(72-33.158)/15 = 3314.5173 Btu/hr
Q = dQ x Hours = 3314.5173 x 24hours/day x 30days/month x 5months = 11.93 mmBtu

c) The natural gas furnace in the house is 90% efficient and natural gas costs $10 /mmBtu (million Btu = mmBtu). If the total heat loss through the walls during the winter was 10 mmBtu/winter, determine the annual cost of the natural gas necessary to offset the heat loss through the walls.
Annual cost = 10 mmBtu/winter /90% x $10/mmBtu = $111.11

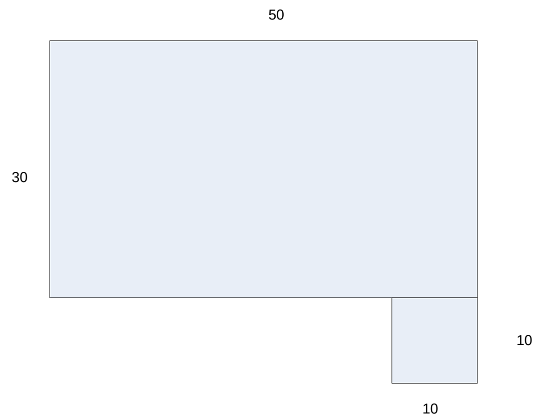
7) (U, G) Consider insulating the exterior wall of a bathroom with fiberglass batt insulation with waterproof facing on one side of the fiberglass batt. Should the waterproof facing be placed outward or inward? Why?
It should be placed inside the insulation, so that water vapor does not diffuse through the insulation and then condense on the exterior side of the insulation.

8) (G) Consider a rectangular one-story house with floor area of 2,000 ft^2. The ratio of the house length L to width W is called the aspect ratio k = L/W. When k = 1 the length and width of the house are the same, the house is square and wall area is minimized. As k increases, the house becomes more rectangular, and wall area increases. Calculate the ratio Awall/Awall,min at aspect ratios of k = 1, 2, 4 and 6. What does this result indicate about wall heat loss from the long, narrow houses?
The height of the house is H, the area of the wall is A.
So, A/H = 2 x (L+W)

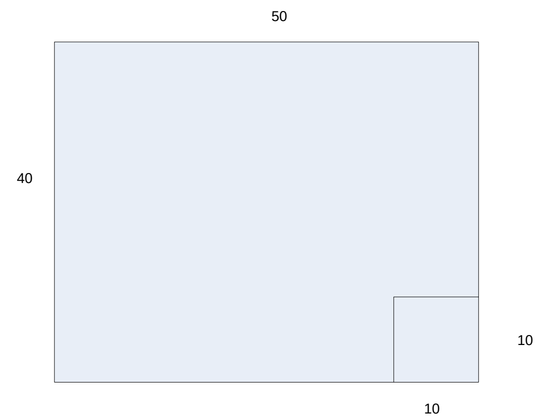
a) k = 1, L = W = 44.72 ft,
   A/H = 178.89
   Awall/Awall = 1
b) k = 2, L = 2 x W, L x W = 2000 ft^2
   A/H = 189.74
   Awall/Awall = 2
c) k = 4, L = 4 x W, L x W = 2000 ft^2
   A/H = 223.61
   Awall/Awall = 4
d) k = 6, L = 6 x W, L x W = 2000 ft^2
   A/H = 255.60
   Awall/Awall = 6
   Awall/Awall, min at ratios of k = 1.

So, wall loss more and more heat from the long, narrow houses.

9) (G) Consider a one-story house with 8 foot high walls designed with a small room that extends from the back of the house as in (A). a) Calculate the ratio of exterior wall area to floor area: Awall/Afloor for (A). Next consider the house if the position of the walls were redesigned to include the small room as in (B). b) Calculate the ratio of exterior wall area to floor area: Awall/Afloor for (B). c) Calculate the percent increase in wall heat loss per floor area of design (A) compared to design (B).



(A)



(B)

a) Awall = (30+50+30+10+10+50) x 8 = 1440 ft^2
   Afloor = 30 x 50 + 10 x 10 = 1600 ft^2
   Awall/Afloor = 0.9

b) Awall = 2x(50+40)x8 = 1440 ft^2
   Afloor = 50 x 40 = 2000 ft^2
   Awall/Afloor = 0.72

c) dQ = A x (Ti-To)/R ➔ dQ ~ A
   0.9 – 0.72 = 0.18 = 18%

10) (G) If you were determining building codes, what minimum R value for walls would you recommend for Minneapolis with average winter air temperature of 20 F and for Phoenix with average winter air temperature of 50 F, if winter lasts 2,500 hours? Why?

11) (U, G) The average outdoor air temperature is 30 F and the average indoor air temperature is 70 F for 2,500 hours during winter. The area of the walls is 1,200 ft^2. The efficiency of the furnace is 90% and the cost of fuel is $10 /mmBtu. Calculate the fuel energy savings from adding R = 10 hr-ft^2-F/Btu insulation to a wall with R= 1 insulation hr-ft^2-F/Btu including convection coefficients. Next calculate the fuel energy savings from adding R = 10 hr-ft^2-F/Btu insulation to a wall with R = 10 hr-ft^2-F/Btu insulation including convection coefficients. What does this example illustrate?

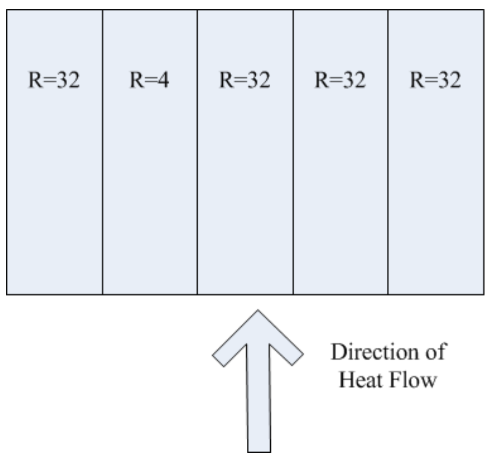
a) R_T = 10+1 =11 hr-ft^2-F/Btu
   dQ = A x (Ti-To)/R = 1,200 ft^2 x (70 F - 30 F) / 1 hr-ft^2-F/Btu = 48,000 Btu/hr
   dQ' = A x (Ti-To)/R_T = 1,200 ft^2 x (70 F - 30 F) / 11 hr-ft^2-F/Btu = 4363.64 Btu/hr

The fuel energy savings = (dQ – dQ') x Hours / efficiency x \$10 /mmBtu = (48,000 – 4363.64) Btu/hr x 2,500 hrs / 90% x 10^{-6} mmBtu/Btu x \$10 /mmBtu = \$1212.12

b) R_T = 10+10 =20 hr-ft^2-F/Btu
dQ = A x (Ti-To)/R = 1,200 ft^2 x (70 F - 30 F) / 10 hr-ft^2-F/Btu = 4,800 Btu/hr
dQ' = A x (Ti-To)/R_T = 1,200 ft^2 x (70 F - 30 F) / 20 hr-ft^2-F/Btu = 2,400 Btu/hr
The fuel energy savings = (dQ – dQ') x Hours / efficiency x \$10 /mmBtu = (4,800 – 2,400) Btu/hr x 2,500 hrs / 90% x 10^{-6} mmBtu/Btu x \$10 /mmBtu = \$66.67

c) The worse the insulation properties of the house, the greater the room can be improved.

12) (G) A wall has 5 panels as shown below. The thermal resistance of four of the panels is R = 32 hr-ft^2-F/Btu including convection coefficients. The contractor forgot to add cavity insulation to one panel and the thermal resistance of that panel, including convection coefficients, is R = 4 hr-ft^2-F/Btu. What fraction of total heat loss occurs through the uninsulated panel? What does this example illustrate?



R_{tot} = 1 / [AF1 / R1 + AF2 / R2 + AF3 / R3 + AF4 / R4 + AF5 / R5] = 1 / [0.2 / 32 + 0.2 / 4 + 0.2 / 32 + 0.2 / 32 + 0.2 / 32] = 13.333
dQ = A x (Ti-To) / Rtot = A x (Ti-To) / 13.333
dQ_{unins} = 0.2 x A x (Ti-To) / R_{unins} = 0.2 x A x (Ti-To) / 4
dQ_{unins} / dQ = 0.667 = 66.7%

Small thermal resistance will lose more heat.