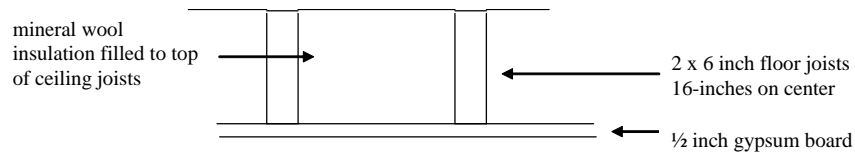


# Energy Efficient Buildings

## Roof Homework

- 1) (U,G) Consider a house with inside air temperature of 72 F. Use outdoor air temperature and solar data from the “day-type” WeaTran output from the Dayton International Airport TMY3 file for July 3 1995 at 15 hours (3 pm). The dimensions of the ceiling and flat roof are 30 ft x 50 ft. The house has an attic with a  $R = 2$  (hr-ft<sup>2</sup>-F/Btu) roof (including convection coefficients on both sides of the roof) with dark shingles ( $\alpha = 0.9$ ). The ceiling construction is shown below.



- a. Calculate the solair temperature (F).

Dayton airport_hr_us - 记事本										
文件(F) 编辑(E) 格式(O) 查看(V) 帮助(H)										
07	03	1995	11	80.06	217	130	74	37	37	0.014
07	03	1995	12	80.96	261	92	89	25	25	0.014
07	03	1995	13	82.04	266	38	96	28	28	0.013
07	03	1995	14	82.94	264	27	92	73	27	0.013
07	03	1995	15	82.94	269	15	78	133	15	0.013
07	03	1995	16	84.02	200	28	87	150	28	0.013
07	03	1995	17	82.04	162	27	31	173	27	0.013
07	03	1995	18	82.04	95	33	33	109	43	0.013
07	03	1995	19	80.06	52	21	21	63	36	0.014
07	03	1995	20	77.00	13	6	6	9	9	0.014
07	03	1995	21	75.02	0	0	0	0	0	0.014
07	03	1995	22	75.02	0	0	0	0	0	0.014

$$T_{oa} = 82.94 \text{ F}$$

$$L_h = 269 \text{ Btu/hr-ft}^2$$

$$\alpha = 0.9$$

$$h_{ext} = 4 \text{ Btu/ht-ft}^2\text{-F}$$

$$T_{sa} = T_{oa} + L_h \alpha / h_{ext} = 82.94 \text{ F} + 269 \text{ Btu/hr-ft}^2 \times 0.9 / 4 \text{ Btu/ht-ft}^2\text{-F} = 143.47 \text{ F}$$

- b. Calculate the R-value of the ceiling (hr-F-ft<sup>2</sup>/Btu) (including convection coefficients).

$$R_{hi} = 0.92 \text{ hr-F-ft}^2\text{/Btu}$$

$$R_{joists} = 5.625 \times 0.833 = 4.686 \text{ hr-F-ft}^2\text{/Btu}$$

$$R_{insulation} = 5.625 \times 3 = 16.875 \text{ hr-F-ft}^2\text{/Btu}$$

$$R_{gypsum} = 0.45 \text{ hr-F-ft}^2\text{/Btu}$$

$$R_{ho} = 0.25 \text{ hr-F-ft}^2\text{/Btu}$$

$$A_{joists} / A_t = 1.625 / 16 = 0.102$$

$$A_{ins} / A_t = 1 - 0.102 = 0.898$$

$$R_{joists \text{ path}} = R_{hi} + R_{joists} + R_{gypsum} + R_{ho} = 6.306 \text{ hr-F-ft}^2\text{/Btu}$$

$$\begin{aligned} R_{\text{insulation path}} &= R_{\text{hi}} + R_{\text{insulation}} + R_{\text{gypsum}} + R_{\text{ho}} = 18.495 \text{ hr-F-ft}^2/\text{Btu} \\ R_{\text{ceil}} &= 1 / [(A_{\text{ins}}/A_{\text{t}})/R_{\text{ins}} + (A_{\text{joists}}/A_{\text{t}})/R_{\text{joists}}] = 15.451 \text{ hr-F-ft}^2/\text{Btu} \end{aligned}$$

- c. Using the simplified method, calculate the total R value (hr-F-ft<sup>2</sup>/Btu) of the roof, attic and ceiling.  
 $R_{\text{roof}} = 2 \text{ hr-ft}^2\text{-F/Btu}$   
 $R_{\text{ceil}} = 15.451 \text{ hr-F-ft}^2/\text{Btu}$   
 $R_{\text{attic}} = 2 \text{ hr-ft}^2\text{-F/Btu}$   
 $R_{\text{t}} = R_{\text{roof}} + R_{\text{ceil}} + R_{\text{attic}} = 2 \text{ hr-ft}^2\text{-F/Btu} + 15.451 \text{ hr-F-ft}^2/\text{Btu} + 2 \text{ hr-ft}^2\text{-F/Btu} = 19.451 \text{ hr-F-ft}^2/\text{Btu}$
- d. Using the simplified method, calculate the heat gain rate (Btu/hr) through the roof/ceiling into the house.  
 $A_{\text{ceil}} = 30 \text{ ft} \times 50 \text{ ft} = 1500 \text{ ft}^2$   
 $Q_{\text{c}} = [A_{\text{ceil}} / (R_{\text{roof}} + R_{\text{attic}} + R_{\text{ceil}})] (T_{\text{sa}} - T_{\text{ia}}) = [1,500 \text{ ft}^2 / (2 + 15.451 + 2) \text{ hr-ft}^2\text{-F/Btu}] (143.47 - 72) \text{ F} = 5,511.542 \text{ Btu/hr}$
- e. The rate of air leakage through the attic is 100 ft<sup>3</sup>/min. Calculate the effective UA (Btu/hr-F) of the air leaking through the attic.  
 $U_{\text{Avent}} = V \times \text{pcp} = 100 \text{ ft}^3/\text{min} \times 60 \text{ min/hr} \times 0.018 \text{ Btu/ft}^3\text{-F} = 108 \text{ Btu/hr-F}$
- f. Using in the attic temperature method, calculate the attic air temperature (F).  
 $U_{\text{Aroof}} = A_{\text{roof}} / R_{\text{roof}} = 1,500 \text{ ft}^2 / 2 \text{ hr-ft}^2\text{-F/Btu} = 750 \text{ Btu/hr-F}$   
 $U_{\text{Aceil}} = A_{\text{ceiling}} / R_{\text{ceil}} = 1,500 \text{ ft}^2 / 15.451 \text{ hr-ft}^2\text{-F/Btu} = 97.08 \text{ Btu/hr-F}$   
 $T_{\text{a}} = [U_{\text{Aroof}} T_{\text{sa}} + U_{\text{Avent}} T_{\text{oa}} + U_{\text{Aceil}} T_{\text{ia}}] / [U_{\text{Aroof}} + U_{\text{Avent}} + U_{\text{Aceil}}]$   
 $= [750 \text{ Btu/hr-F} \times 143.47 \text{ F} + 108 \text{ Btu/hr-F} \times 82.94 \text{ F} + 97.08 \text{ Btu/hr-F} \times 72 \text{ F}] / [750 \text{ Btu/hr-F} + 108 \text{ Btu/hr-F} + 97.08 \text{ Btu/hr-F}] = 129.36 \text{ F}$
- g. Using in the attic temperature method, calculate the rate of heat gain (Btu/hr) into the house.  
 $Q_{\text{c}} = U_{\text{Aceil}} (T_{\text{a}} - T_{\text{ia}}) = 97.08 \text{ Btu/hr-F} \times (129.36 \text{ F} - 72 \text{ F}) = 5,568.57 \text{ Btu/hr}$
- h. Using in the attic temperature method, calculate the rate of heat gain (Btu/hr) into an energy-efficient house if the roof has light colored shingles with ( $\alpha = 0.4$ ), attic ventilation is increased to 200 ft<sup>3</sup>/min and the ceiling has an *additional* 6-inches of loose mineral wool insulation sprayed on top of the existing ceiling.  
 $T_{\text{oa}} = 82.94 \text{ F}$   
 $I_{\text{h}} = 269 \text{ Btu/hr-ft}^2$   
 $\alpha = 0.4$   
 $h_{\text{ext}} = 4 \text{ Btu/ht-ft}^2\text{-F}$

$$T_{sa} = T_{oa} + \frac{I_h \alpha}{h_{ext}} = 82.94 \text{ F} + \frac{269 \text{ Btu/hr-ft}^2 \times 0.4}{4 \text{ Btu/ht-ft}^2\text{-F}} = 109.84 \text{ F}$$

$$R_{ceil}' = R_{ceil} + R_{add \text{ ins}} = 15.451 \text{ hr-F-ft}^2/\text{Btu} + 6 \times 3 \text{ hr-F-ft}^2/\text{Btu} = 33.451 \text{ hr-F-ft}^2/\text{Btu}$$

$$R_t' = R_{roof} + R_{ceil}' + R_{attic} = 2 \text{ hr-ft}^2\text{-F}/\text{Btu} + 33.451 \text{ hr-F-ft}^2/\text{Btu} + 2 \text{ hr-ft}^2\text{-F}/\text{Btu} = 37.451 \text{ hr-F-ft}^2/\text{Btu}$$

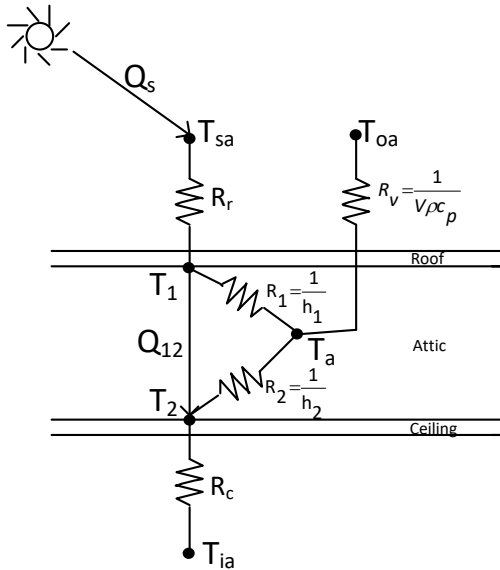
$$U_{Avent}' = V' \times pcp = 200 \text{ ft}^3/\text{min} \times 60 \text{ min/hr} \times 0.018 \text{ Btu/ft}^3\text{-F} = 216 \text{ Btu/hr-F}$$

$$U_{Aceil}' = A_{ceiling} / R_{ceil}' = 1,500 \text{ ft}^2 / 33.451 \text{ hr-ft}^2\text{-F}/\text{Btu} = 44.84 \text{ Btu/hr-F}$$

$$T_a' = \frac{[U_{Aroof} T_{sa} + U_{Avent} T_{oa} + U_{Aceil}' T_{ia}]}{[U_{Aroof} + U_{Avent} + U_{Aceil}']} = \frac{[750 \text{ Btu/hr-F} \times 109.84 \text{ F} + 216 \text{ Btu/hr-F} \times 82.94 \text{ F} + 44.84 \text{ Btu/hr-F} \times 72 \text{ F}]}{[750 \text{ Btu/hr-F} + 216 \text{ Btu/hr-F} + 44.84 \text{ Btu/hr-F}]} = 102.41 \text{ F}$$

$$Q_c' = U_{Aceil}' (T_a' - T_{ia}) = 44.84 \text{ Btu/hr-F} \times (102.41 \text{ F} - 72 \text{ F}) = 1,363.58 \text{ Btu/hr}$$

- 2) (G) Consider the attic radiation model shown below. 1 represents the underside of the roof and 2 represents the top of the ceiling. Use the following input data:



$Q_s = 300$  "Btu/hr-ft<sup>2</sup>"  
 $A_1 = 100$  "ft<sup>2</sup>"  
 $A_2 = 100$  "ft<sup>2</sup>"  
 $T_{ia} = 72$  "F"  
 $T_{oa} = 88$  "F"  
 $h_1 = 1/.92$  "Btu/hr-ft<sup>2</sup>-F"  
 $h_2 = 1/.92$  "Btu/hr-ft<sup>2</sup>-F"  
 $h_{ext} = 4$  "Btu/hr-ft<sup>2</sup>-F"  
 $R_r = 2$  "hr-ft<sup>2</sup>-F/Btu"  
 $R_c = 12$  "hr-ft<sup>2</sup>-F/Btu"  
 $f_{12} = 1.0$  "view factor"  
 $pcp = 0.018$  "Btu/ft<sup>3</sup>-F"  
 $e_2 = 0.80$  "emissivity"

- a) Consider a base case with a dark roof (alpharroof = 0.80), a non-reflective underside of the roof (e1 = 0.85), and limited natural air flow through the attic (Va = 60 cfm). Calculate the heat flux into the room (Btu/hr).

Tsa	148			
T1	116.49	-7.1E-05	0	
T2	109.96	7.1E-05	0	
Ta	107.43	3.77E-11	0	
Q12	591.09	<b>Qc</b>	<b>316.33</b>	
Qs	300	alpharroof	0.8	
A1	100	e1	0.85	
A2	100	Va	3600	60*60
Tia	72			
Toa	88			
h1	1.086956522			
h2	1.086956522			
hext	4			
Rr	2			
Rc	12			
f12	1			
pcp	0.018			
e2	0.8			
Stephan Boltzman	1.714E-09			

- b) Next consider the energy-efficient case with light-colored shingles (alpharroof = 0.3), a low-emissivity radiant barrier on the underside of the roof (e1 = 0.1) and an attic ventilation fan (Va = 500 cfm). Calculate the heat flux into the room (Btu/hr).

Tsa	111			
T1	95.39	1.26E-05	0	
T2	88.58	-1.3E-05	0	
Ta	89.14	-1.6E-09	0	
Q12	76.63	Qc	138.14	
Qs	300	alpharroof	0.3	
A1	100	e1	0.1	
A2	100	Va	30000	500*60
Tia	72			
Toa	88			
h1	1.086956522			
h2	1.086956522			
hext	4			
Rr	2			
Rc	12			
f12	1			
pcp	0.018			
e2	0.8			
Stephan Boltzman	1.714E-09			

- 3) (G) Calculate the hourly steady-state and transient heat fluxes (Btu/hr-ft<sup>2</sup>) into a building through a 12"inch green roof on top of a R = 8 traditional roof. Show your results graphically. Report the peak steady state and transient heat fluxes (Btu/hr-ft<sup>2</sup>) after the effect of initial conditions becomes negligible. Also report the delay (hrs) between the peak steady state and transient heat fluxes after the effect of initial conditions becomes negligible. Assume the solair temperature varies sinusoidally over a 24-hour period about a mean temperature of 85 F with an amplitude of 12 F. Soil properties are:

$$\rho = 109 \text{ lb/ft}^3$$

$$c_p = 0.191 \text{ Btu/lb-F}$$

$$R_{\text{soil}} = 1.846 \text{ hr-ft}^2\text{-F/Btu per ft}$$

$$R_s = R_{\text{soil}} \times dx / 2 = 1.846 \text{ hr-ft}^2\text{-F/Btu per ft} \times 0.5 \text{ ft} = 0.924 \text{ hr-ft}^2\text{-F/Btu}$$

$$R_{\text{sr}} = R_{\text{roof}} + R_{\text{sr}} = (8 + 0.924) \text{ hr-ft}^2\text{-F/Btu} = 8.924 \text{ hr-ft}^2\text{-F/Btu}$$

Hourly values of Tsa over a 24-hour period are calculated as:

$$T_{\text{sa}} = T_{\text{sa,mean}} + T_{\text{sa,amp}} \times \sin(2 \pi \text{ hr} / 24)$$

For  $hr = 1$ ,  $T_s$  is set to 70 F as an initial condition. In all subsequent hours,  $T_s^+$ , which is soil temperature at the end of the hour, is calculated from:

$$T_s^+ = T_s + dt / (dx \rho cp) [ (T_{sa} - T_s)/R_s + (T_{ia} - T_s)/R_{sr} ] =$$

After  $T_s$  is known, the transient and steady state heat fluxes into the building are calculated as:

$$Q = (A/R_{sr}) (T_s - T_{ia})$$

$$Q_{ss} = A / (R_s + R_{sr}) (T_{sa} - T_{ia})$$

